17. Using Annihilator method, find the particular solution of the problem $y'' - 3y' + 2y = x^2$.

18. Let $\phi_1, \phi_2, \ldots \phi_n$ be n solutions of $L(y) = 0$ on an interval I. and let x_0 be any point in I. Then prove that $W(\phi_1, \phi_2 \ldots, \phi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1(t)\,dt\right] W(\phi_1, \phi_2, \ldots \phi_n)(x_0)$.

19. Derive the Bessel equation.

20. Discuss the necessary and sufficient condition for the equation $M(x,y)dx + N(x,y)dy = 0$ to be exact.

APRIL/MAY 2024

23PMA13 — ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts & Science College T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. When two functions are said to be linearly dependent on I?

2. Show that $\varphi_1(x) = x^2, \varphi_2(x) = 5x^2$ are linearly dependent.

3. Define characteristic polynomial.

4. Define Wronskian function.

5. Define linear space of functions.

6. State the existence theorem for analytic coefficients.

7. What are regular singular points?

8. Find the singular point of $xy'' + 4y = 0$.

9. Inspect whether the equation $x^2y^3dx - x^3y^2dy = 0$ is exact or not.

10. When do we say that a function 'f' satisfies a Lipschitz condition?

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Find all the solutions of the equation $y'' - 4y = 0$.

Or

(b) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0, then prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$.

12. (a) Let ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = 0$ on an interval I containing a point x_0. Then show that for all x in I, $\|\phi(x_0)\|e^{-k|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\|e^{k|x-x_0|}$ where $k = 1 + |a_1| + \ldots + |a_n|$.

Or

(b) Find the solution of $y^{(4)} + y = 0$.

13. (a) Prove that "There exist n linearly independent solutions of $L(y) = 0$ on I".

Or

(b) State and prove uniqueness theorem for linear equations with variable coefficients.

14. (a) Derive the indicial polynomial of the equation $x^2y'' + xy' + y = 0$.

Or

(b) Find the roots of the equation $x^2y'' + \frac{3}{2}xy' + xy = 0$

15. (a) Find the real valued solution of the equation $y' = x^2y$.

Or

(b) Check whether the equation $2xydx + (x^2 + 3y^2)dy = 0$ is exact and solve it.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that "Two solutions ϕ_1 and ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.